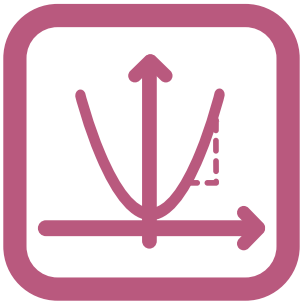


Basic Maths
Course booklet

Algebra, precalculus and calculus for college and university students. Contains topics ranging from numbers to differentiation and intergration

SOWISO

ABOUT & PRICING

About SOWISO

SOWISO offers:

- a homework, practice and **learning environment**;
- personalised **feedback** on all answer attempts;
- different **testing and assessment** tools;
- customisable **mathematics courses** with explanations, examples, and endless **randomised practice exercises**;
- an authoring tool to **create original material**;
- **learning analytics** giving detailed insight into student performance;
- **integration** with your LMS/VLE.

Our learning environment guides students along as they solve problems. When doing exercises, students can enter open answer calculations or mathematical formulas. The software will analyse their answer and provide targeted feedback and hints helping the student understand the next step in the solution process, and/or highlight any mistakes they made.

SOWISO increases student engagement and saves teachers time checking and grading!

Pricing

SOWISO partners with higher education institutions on a SAAS licensing basis.

The cost for the platform starts at € 5.50 per student per year, with an additional per student per year fee of € 7.50 per course.

A second licensing model is one in which students pay for their own license in our webshop.

Our digital courses are a fully interactive alternative for paper books and offer a personalised and adaptive learning experience that fits today's generation of students.

How are courses structured?

The courses are structured in chapters and subchapters consisting of units. The unit subjects are listed in more detail on the following pages.

Each unit consists of (at least) one theory page and one package of exercises.

Theory pages contain explanations, (randomised) examples and visualisations and (interactive) graphs.

The packages of **exercises** contain on average around 10 exercises. Each of these exercises are randomised, allowing for endless practicing, and include targeted hints and personalised feedback for the students while solving the exercises.

COURSE CONTENT

Chapter 1: Numbers (17 topics)

1. *Integers (3 topics)*
    a. Calculating with integers
    b. Integers
    c. Division of integers

2. *Negative numbers (1 topic)*
    a. Absolute value

3. *Fractions (7 topics)*
    a. Fractions
    b. Equivalent fractions
    c. Simplifying fractions
    d. Addition and subtraction of fractions
    e. Multiplication and division of fractions
    f. Integer powers of fractions
    g. Decimal numbers

4. *Powers and roots (6 topics)*
    a. Exponents
    b. Calculating with powers
    c. Roots of integers
    d. Roots of fractions
    e. Standard notation of higher roots
    f. Order of operations for powers and roots

Chapter 2: Algebra (27 topics)

5. *Variables (5 topics)*
    a. Variables
    b. Sum and product of variables
    c. Substitution
    d. Simplification
    e. Simplification with algebraic rules

PRACTICE & THEORY EXAMPLE

Quadratic equations: Solving quadratic equations

Solving quadratic equations by factorization

Solving the following equation by means of factorization can be done in steps. Drag the steps in the correct order.

$$-2 \cdot t^2 - 22 \cdot t + 63 = 79 - 10 \cdot t$$

The center column shows the steps in words, the right column shows the equation after the step is applied.

step 1	left hand side factorized	$t + 2 = 0 \vee t + 4 = 0$
step 2	$A \cdot B = 0$ gives $A = 0 \vee B = 0$	$-2 \cdot t^2 - 12 \cdot t - 16 = 0$
step 3	constant terms to the right hand side	$t^2 + 6 \cdot t + 8 = 0$
step 4	reduce to 0	$(t + 2) \cdot (t + 4) = 0$
step 5	divided by -2	$t = -2 \vee t = -4$

Practise example

Trigonometry: Angles with sine, cosine and tangent

Angles in radians

So far we have expressed angles in degrees, but in mathematics angles are often expressed in radians. To introduce radians we will use a circle with radius 1. We call this the unit circle.

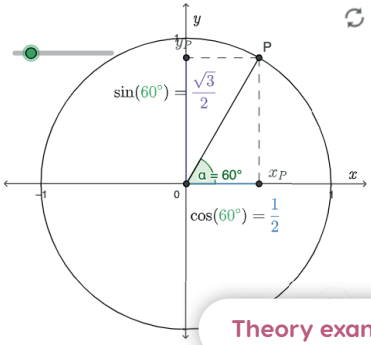
Unit circle

The **unit circle** is a circle with origin center $[0, 0]$ and radius 1.

The point $P = [x_P, y_P]$ starts at $[1, 0]$ and moves counterclockwise across the unit circle. The **angle of rotation** is called α.

Therefore $\sin(\alpha) = y_P$ and $\cos(\alpha) = x_P$.

In this way we can also define angles greater than $90°$ degrees with the sine and the cosine.



Theory example

6. *Calculating with exponents and roots (8 topics)*
    a. Integer powers
    b. Calculating with integer exponents
    c. Positive integer exponents
    d. Square roots
    e. Calculating with square roots
    f. Higher degree roots
    g. Calculating with fractional exponents
    h. Order of operations

7. *Expanding brackets (2 topics)*
    a. Expanding brackets
    b. Expanding double brackets

8. *Factorization (2 topics)*
    a. Factoring out
    b. Factorization

9. *Notable products (2 topics)*
    a. The square of a sum or a difference
    b. The difference of two squares

10. *Adding and subtracting fractions (8 topics)*
    a. Fractions
    b. Simplifying fractions
    c. Addition and subtraction of like fractions
    d. Making fractions similar
    e. Addition and subtraction of fractions
    f. Multiplication of fractions
    g. Division of fractions
    h. Fraction decomposition

Chapter 3: Linear formulas and equations (13 topics)

11. *Formulas (3 topics)*
    a. Formulas
    b. Dependent and independent variables
    c. Graphs

THEORY & EXERCISE EXAMPLE

Quadratic equations: Parabola

Parabola

Graph

The graph of a quadratic

$$y = ax^2 + bx + c$$

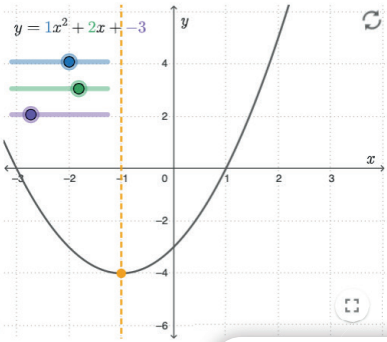
is called a **parabola.**

If $a > 0$ the graph is an **upward opening parabola.**

If $a < 0$ the graph is a **downward opening parabola.**

An upward opening parabola has a minimum and downward opening parabola has a maximum. In both cases, this point is referred to as the **vertex** of the graph.

The parabola is symmetrical about the vertical line through the top of the graph. Such a line is also called a **line of symmetry**.



Theory example

Algebra: Adding and subtracting fractions

Addition and subtraction of fractions

Put over a common denominator, expand all brackets in the numerator and simplify as much as possible:

$$\frac{p}{p-8} + \frac{1}{p+8}$$

Hint
First put the fractions over the same denominator.

$$\frac{p}{p-8} + \frac{1}{p+8} = \quad —$$

Check

Theory | Solution | Hint

Practise example

12. *Linear functions (4 topics)*
    a. Linear formula
    b. Slope and intercept
    c. Composing a linear formula
    d. Parallel and intersecting linear formulas

13. *Linear equations and inequalities (6 topics)*
    a. Linear equations
    b. The general solution of a linear equation
    c. Intersection points of linear formulas with the axes
    d. Intersection point of two linear formulas
    e. Linear inequalities
    f. General solution of a linear inequality

Chapter 4: Systems of linear equations (8 topics)

14. *An equation of a line (4 topics)*
    a. A linear equation with two unknowns
    b. Solution of linear equations with two unknowns
    c. The equation of a line
    d. Composing the equation of a line

15. *Two equations with two unknowns (4 topics)*
    a. Systems of linear equations
    b. Solving systems of linear equations by substitution
    c. Solving systems of equations by elimination
    d. General solution system of linear equations

Chapter 5: Quadratic equations (13 topics)

16. *Parabola (2 topics)*
    a. Quadratics
    b. Parabola

17. *Solving quadratic equations (4 topics)*
    a. Quadratic equations
    b. Solving quadratic equations by factorization
    c. Solving quadratic equations by completing the square
    d. The quadratic formula

Differentiation: Applications of derivatives

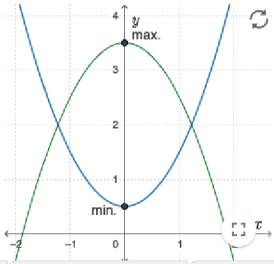
Extreme values

Maxima and minima

The highest value of a part of a graph is called a **local maximum**.

The lowest value of a part of a graph is called a **local minimum**.

Both are **extreme values** of a function.



Extreme values on restricted domain | Extrema are function values | Global maxima and minima

Using the derivative, we can easily calculate the extreme values of a function.

Extreme value

If a function $f(x)$ has a local maximum or minimum at $x = c$ then $f'(c) = 0$.

Example

$$\begin{aligned} f(x) &= x^2 \\ f'(x) &= 2x \\ f'(0) &= 0 \end{aligned}$$

Differentiable

Calculating extreme values

Step-by-step

Determine the extreme values of a function $f(x)$. Determine for each extreme value whether it is a local minimum, local maximum or neither.

Example

$$f(x) = x^4 - 2x^2$$

Step 1 Calculate the derivative $f'(x)$.

$$f'(x) = 4x^3 - 4x$$

Step 2 Solve $f'(x) = 0$ to find the x coordinates of the points which are possibly an extreme value.

$$\begin{aligned} 4x^3 - 4x &= 0 \\ x &= 0 \vee 4x^2 - 4 = 0 \\ x &= 0 \vee x^2 = 1 \\ x &= 0 \end{aligned}$$

Theory example

18. *Drawing parabolas (4 topics)*
    a. Intersection of parabolas with the axes
    b. Vertex of a parabola
    c. Drawing of parabolas
    d. Transformations of parabolas

19. *Intersection points of parabolas (2 topics)*
    a. Intersection points of a parabola with a line
    b. Intersection points of parabolas

20. *Quadratic inequalities (1 topics)*
    a. Quadratic inequalities

Chapter 6: Functions (25 topics)

21. *Domain and range (5 topics)*
    a. Function and formula
    b. Function rule
    c. Intervals
    d. Domain
    e. Range

22. *Power functions (3 topics)*
    a. Power functions
    b. Transformations of power functions
    c. Equations with power functions

23. *Higher degree polynomials (5 topics)*
    a. Polynomials
    b. Equations with polynomials
    c. Solving higher degree polynomials with factorization
    d. Solving higher degree polynomials with the quadratic equation
    e. Higher degree inequalities

THEORY & EXERCISE EXAMPLE

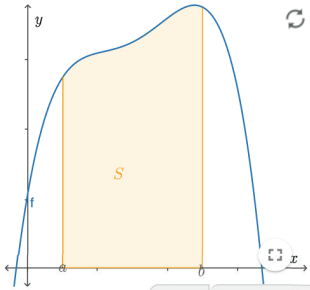
Integration: The definite integral

Area

Statement

The area of the surface V above the x-axis and bound by the graph of f, the lines $x = a$ and $x = b$ is equal to:

$$\int_a^b f(x)\,\mathrm{d}x$$



Proof | Application in physics

We have now seen how to calculate the area of a surface above the x-axis, but in the same manner we can calculate a surface below the x-axis.

Statement

Theory example

Linear formulas and equations: Linear equations and inequalities

Linear equations

Find the unique value of x for which $8 \cdot x - 6 = -6$ is true.

Give your answer in the form $x = \ldots$ and simplify as much as possible.

Hint
Remember to first subtract -6 on both sides of the equation.

$8 \cdot x = -6 - 6$	✗	No, on the right-hand side, you have subtracted 6, but you should have added it.
$8 \cdot x = -6 + 6$	✓	Eliminate each of the additions and subtractions on the right.
$x = 0$	✓	Correct answer

Practise example

24. *Power functions and root functions (5 topics)*
    a. Root functions
    b. Transformations of root functions
    c. Root equations
    d. Solving root equations with substitution
    e. Inverse functions

25. *Fractional functions (7 topics)*
    a. Asymptotes and hyperbolas
    b. Power functions with negative exponents
    c. Transformations of power functions with negative exponents
    d. Linear fractional functions
    e. Linear fractional equations
    f. Inverse of linear fractional functions
    g. Quotient functions

Chapter 7: Exponential functions and logarithms (13 topics)

26. *Exponential functions (3 topics)*
    a. The exponential function
    b. Exponential equations
    c. Transformations of the exponential function

27. *Logarithmic functions (10 topics)*
    a. The logarithmic function
    b. Logarithmic equations
    c. Exponential equations
    d. Isolating variables
    e. Rules for logarithms
    f. More logarithmic equations
    g. Change of base
    h. Solving equations using substitution
    i. Graph of logarithmic functions
    j. Transformations of the logarithmic function

Chapter 8: Trigonometry (12 topics)

28. *Angles with sine, cosine, and tangent (8 topics)*
    a. Angles
    b. Triangles
    c. Rules for fight-angled triangles
    d. Angles in radians
    e. Symmetry in the unit circle
    f. Special values of trigonometric functions
    g. Addition formulas for trigonometric functions
    h. Sine & cosine rules

29. *Trigonometric functions (4 topics)*
    a. Trigonometric functions
    b. Transformations of trigonometric functions
    c. Inverse trigonometric functions
    d. Trigonometric equations

Chapter 9: Differentiation (20 topics)

30. *The derivative (4 topics)*
    a. The differene quotient
    b. The difference quotient at a point
    c. The tangent line
    d. The notion of derivative

31. *The derivative of power functions (1 topic)*
    a. The derivative of power functions

32. *Sum and product rule (2 topics)*
    a. The sum rule
    b. The product rule

33. *Chain rule (2 topics)*
    a. Composite functions
    b. The chain rule

34. *The derivative of standard functions (4 topics)*
    a. The derivative of trigonometric functions
    b. The base e and the natural logarithm
    c. The derivative of the natural logarithm
    d. The derivative of exponential functions and logarithms

35. *The quotient rule (1 topic)*
    a. The quotient rule

36. *Applications of derivatives (6 topics)*
    a. Increasing and decreasing
    b. Extreme values
    c. The second derivative
    d. Types of increasing and decreasing
    e. Inflection points
    f. Higher order derivatives

Chapter 10: Integration (17 topics)

37. *Antiderivatives (5 topics)*
    a. The antiderivative of a function
    b. The antiderivative of a power function
    c. Rules of calculation for antiderivatives
    d. Antiderivatives of known functions
    e. Antiderivatives and the chain rule

38. *The definite integral (5 topics)*
    a. Definite integral
    b. Area
    c. Area of a surface between curves
    d. Area between curves
    e. Solid of revolution

39. *Integration techniques (7 topics)*
    a. Substitution method
    b. Trigonometric integrals
    c. Integration by parts
    d. Repeated integration by parts

e. Known antiderivatives of some quotient functions
f. Long division with polynomials
g. Finding the antiderivatives of quotient functions

Missing something? SOWISO allows teachers to create their own content in our authoring environment.

THEORY EXAMPLE

Linear formulas and equations: Linear equations and inequalities

Intersection point of two linear formulas

We have seen how to solve a linear equation. With this same technique we can also determine the coordinates of the intersection points of two linear formulas.

Example

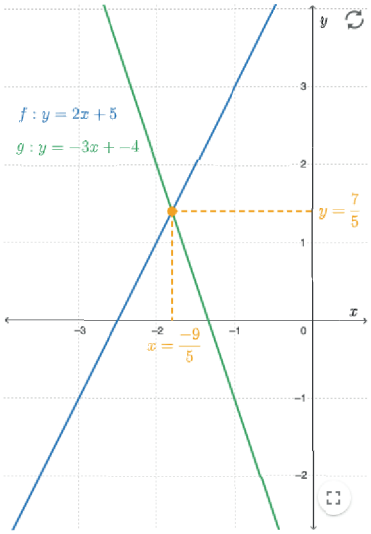
We consider the linear formulas $f : y = 2 \cdot x + 5$ and $g : y = -3 \cdot x - 4$. We can find the x-coordinate of the intersection point by solving the equation $2 \cdot x + 5 = -3 \cdot x - 4$. This is done in the following manner:

$$\begin{aligned} 2 \cdot x + 5 &= -3 \cdot x - 4 && \text{the equation} \\ 5 \cdot x + 5 &= -4 && \text{both sides plus } 3 \cdot x \\ 5 \cdot x &= -9 && \text{both sides minus } 5 \\ x &= -\frac{9}{5} && \text{both sides divided by } 5 \end{aligned}$$

Hence, the x-coordinate of the intersection point is $x = -\frac{9}{5}$.
We can find the y-coordinate by substituting $x = -\frac{9}{5}$ in one of the formulas. This gives us:

$$y = 2 \cdot -\frac{9}{5} + 5 = \frac{7}{5}$$

Hence, the coordinates of the intersection point are $\left[-\frac{9}{5}, \frac{7}{5}\right]$.



Theory example

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